# USING GRAPH THEORY IN LOGISTICS FROM A 21ST CENTURY PERSPECTIVE

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**Abstract:**

This paper describes the basic possibilities of applying graph theory in regional transport based on a change of approach to the customer-carrier relationship. It shows the possibilities of applying graph theory in solving transport problems with a focus on customer requirements. In particular, it focuses on the distribution of demands on the transport network under study, aiming to find the optimal model for the movement and servicing of all nodes for a given number of servicing vehicles, capacity constraints or other specified criteria. The solution to this problem is made by analysing the method of vehicle routing problem and its several modifications depending on the nature of the problem and the framework requirements for solving the problem with subsequent application to selected examples.

**Keywords:** graph theory, VRP, logistics,

## Introduction

Regional sometimes also city logistics is a very young but rapidly developing field. Logistics deals with the management of the flow of goods, materials and services associated with their transportation to the end customer. It starts from the planning of the production of a certain quantity of goods to the delivery of the goods to the end customer. Recently, cost and on-time delivery are the most important factors in logistics. [13]

The rapid development of regions (cities) and the high degree of urbanisation cause problems in the transport of goods, which are dealt with by so-called urban logistics. It has to take into account the constraints caused not only by the customer and the production process but also by the spatial distribution of the region (city). It minimises the impact of transport on the environment and the population. As a consequence of the development of the economy and industry in a given region, the demand for freight and other transport is increasing.

Graph theory deals with the modelling of transportation relationships, and with the help of these models, we can design efficient solutions to the problems of transporting goods in regional areas.

The history of City logistics is relatively short. It originated in the early 1970s when the research was carried out on urban freight transport and the flow of goods in cities. The term "City logistics" first appeared in German texts in the 1980s.

The City Logistics concept solves many complicated logistics problems. Taniguchi et al. (1999) defined City Logistics as "a process for the complete optimization of logistics and transportation activities of private companies in regions, taking into account the traffic environment, traffic congestion, and energy consumption in a market economy". This could be also described as "City Logistics aims to globally optimise logistics systems in the region, taking into account the costs and benefits of the schemes for both the public and the private sector" [9].

City logistics aims to mitigate the effects of freight transport through measures that include the promotion of advanced information and communication technologies, intelligent transport systems (ITS) and various restrictions (congestion control, road pricing, lorry entry restrictions, etc.). This policy aims to improve the traffic flow and the environment of the city.

The challenges and opportunities for improving regional logistics are constantly changing, influenced by factors such as:

- Population growth and urbanization,
- The growth of e-commerce,
- The desire for speed,
- The shift from business models such as C2C (consumer to consumer) to B2B (business to business),
- Climate change and the sustainability of freight growth lead to air and noise pollution, traffic accidents and increased greenhouse gas emissions. [12]

Several approaches and methods can be used in terms of spatial and transport planning and urban development, one of the planning optimization options is the use of methods based on graph theory.

## Graph theory applicable to city logistics challenges

City logistics problem solving helps us find graph theories, using them to model the transportation network where we look for the best solution. Among the basic options, we can include the Clark-Wright algorithm - Method for determining roundabouts [4] - A given algorithm is often used as the basis for more complex algorithms that solve combination problems or problems with certain constraints.

The Truck Dispatching Problem is one of the first papers published on the vehicle steering problem it compares this issue to a Traveling Salesman Problem (TSP) which is known by imposing a di to each point plus adding truck capacity to limit the number of the visited points. [14] The main purpose of a vehicle routing problem (VRP) is to design least cost routes assigned to the vehicles that distribute product to a group of customers; with the characteristic that each route starts and finishes at the same departure point. This type of problem is treated as a classic problem of combinatorial optimization and due to its complexity, is cataloged in the literature as a NP-Hard problem. [15]

Selected problems in graph theory adapted to logistics may be formulated in several methods described below.

**Vehicle Routing Problem** (VRP) - The problem known as Vehicle Routing Problem deals with the transport of goods between the depot and the end customer. The VRP solution is to establish routes where each is served by a single vehicle that starts and ends at a single depot so that all customer requirements are met, costs are minimised and all given constraints are met. VRP can be used to evaluate measures and optimise routes. The result is to use the fewest number of vehicles necessary to meet customer demand, serve the shortest route and reduce the journey time of empty vehicles. This reduces the strain on the transport network and the environmental impact of transport, provides a more comfortable environment for residents and faster transport of goods for customers. A transport network for goods transport is described by a graph where the edges represent the roads connecting the pairs of vertices considered and the vertices are the transport nodes (depot or customer). The edges can be oriented or non-oriented depending on whether a vehicle can travel in both directions or not (e.g. streets with one-way traffic - characteristic of large cities). Each edge is characterized by a value that indicates its length and the time to cross it.

**CVRP (Capacited VRP)** is the basic version of VRP - All customers are served from one depot and requirements are deterministic or known in advance. Vehicles are identical and have a single central depot, only capacity constraints are mandatory. The goal is to minimize the total transportation costs (vehicle costs, cost per km travelled, etc.).

**Vehicle Routing Problem with Time Windows** (VRPTW) is another subtask of the CVRP, looking for optimal routes that meet all customer requirements and do not exceed specified constraints. Each customer is associated with a time window that limits the amount of time a vehicle has to serve the customer. Hard time windows will not allow a vehicle to arrive at a customer later than the lower allowed time limit.

**Vehicle Routing Problem with Soft Time Windows** (VRPSTW) allows carriers to exceed the established time limit with some penalties. That is, if a vehicle arrives earlier, it must wait for the specified time $a$i, otherwise, if it arrives later, it can start but will be fined.

**Vehicle Routing Problem with Backhauls** (VRPB) is a subtask of CVRP, where the set of customers is divided into two subsets. The first is the subset of customers to whom we need to deliver goods. The second is the set of customers from whom we need to pick up the goods. The VRPB problem is to determine the set of routes/destinations/districts that a vehicle must traverse while satisfying given conditions and constraints.

**Vehicle Routing Problem with Pickup and Delivery** (VRPPD) is a subtask of CVRP. The task assignment specifies the transportation requirements. Each requirement is defined by a pickup point, a delivery point, and the transportation requirements between them. Transportation requirements may include goods or people. The objective is to minimize the cost function.

**VRPPD with Time Windows is a generalization of VRPTW**. The time constraint specifies the time intervals during which each stop must be serviced, otherwise, it will cause complications for users and restrictions for passengers. Additionally, for time windows to ensure that each stop is served (satisfied).

**Two-Eschelon Vehicle Routing Problem** (2E-VRP) - a two-level vehicle routing problem [10]. For larger service areas or in urban environments, it is often convenient to extend the VRP problem to two levels and serve the area with transfer points where we can perform the unloading and loading of goods and combine them into vehicles.

**Location-Routing Problem** (LRP) - LRP combines two basic problems:

- Facility Location Problem (FLP) - the problem of the location of a facility (depot, warehouse, factory, etc.),
- Vehicle routing problem (determining routes for operators).

To solve the problem addressed it is possible to use exact, classical heuristics or metaheuristics methods.

**Exact problem-solving methods** provide proof that the solution obtained is optimal. These methods often work on the principle of tree search algorithms.

- Branch and bound method - divides the problem space into smaller subsets that form tree branches and explores them sequentially,
- Branches and cuts method - we have a set of possible solutions and we shift the upper and lower bounds of the constraints, thus reducing the region of possible solutions,
- Branch and Price Method.

**Classical heuristics** - unlike exact methods, they do not ensure finding the best solution. It finds a solution that meets the requirements. The main advantage is the speed of the solution found.

**Metaheuristic** - Represents a class of approximate algorithms that provides an acceptable solution. It can be adapted to different problems. Unlike classical heuristics, there is a possibility to leave the optimum in certain circumstances that there is a possibility of finding a better solution in another possible solution: Metaheuristics use a trade-off between randomization and local search for this purpose.

## Examples of application of the described methods

The first example is the application of the **VRPTW method** on a network that is formed by links representing the communications of the considered transport infrastructure and nodes that represent the location of the distribution depot (node 0) and the customers to be served (nodes 1,2,3 and 4). The graph is time-oriented, with the nearest neighbour being served.

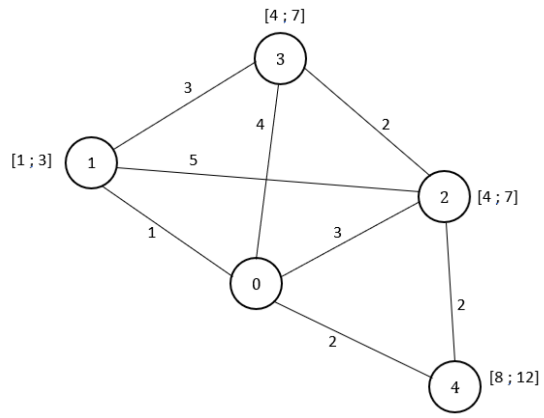


Fig. 1 Location of depot and customers - VRTPW

The following figure shows a possible solution consisting in finding the optimal route for handling requests (serving customers) with the condition of the minimum time consumption function (or minimum length), which is formed by the sum of the time consumption of all routes between the depot and the customers and the final return to the depot. The optimal route in this case is formed in the sequence 0-1-3-2-4-0.

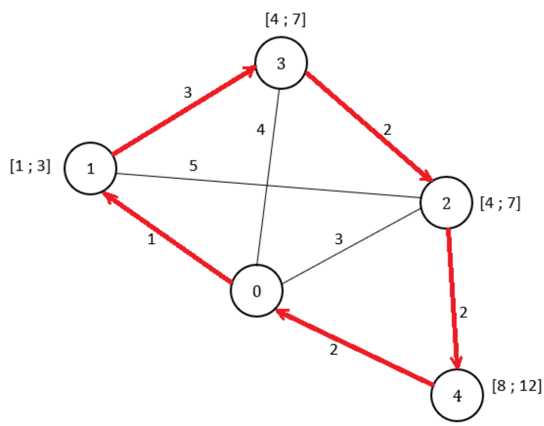


Fig. 2 Example Solution - VRPTW

The second example is the application of the **VRPB method.** Considered is a road or railway infrastructure network which in compliance with graph theory is described as a set of nodes and links. Blue nodes represent individual customers the ones we have to pick up the goods from (backhauls), and the yellow ones are the ones we have to deliver the goods to (linehaul). The values of the individual customers (nodes) were defined as follows - 1=500, 2=700, 3=500, 4=400, 5=800, 6=300, which creates the supply and demand requirements and the vehicle capacity, which is the capacity constraint, was defined by the value 1600. The D is the depot.

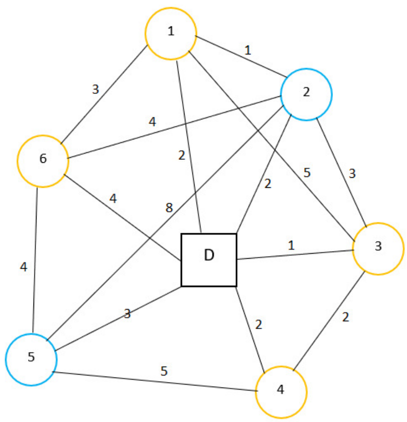


Fig. 3 Road or rail network and customer location - VRPB

The following figure provides a possible solution to the problem to satisfy the requirements of all customers under the defined vehicle capacity constraints, and the resulting number of vehicles needed to serve the study area. The optimal route for handling requests (serving customers) with the condition of minimum of time consumption function (or minimum length), which is formed by the sum of the time consumption (or length) of all routes between the depot and the customer is as follows. To operate the given solution we need 2 vehicles, the total length of the circuit is 24 units (4 + 3 + 1 + 8 + 3 + 1 + 2 + 2). The arrows define the direction and connection of each node of the graph.

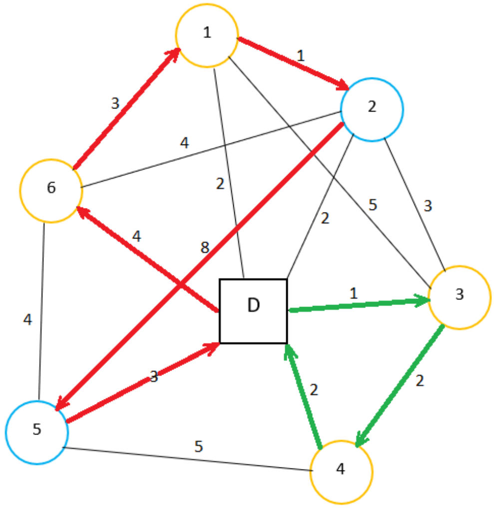


Fig. 4 The first variant of the solution - VRPB

To perform the second possible solution we also need 2 vehicles to operate the given network, the total length of the circuit is also 24 units (2 + 3 + 4 + 8 + 2 + 1 + 2 + 2).

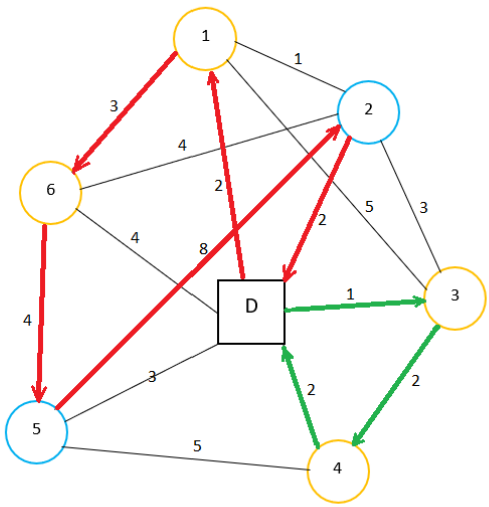


Fig. 5 The second variant of the solution

Given two solutions (Fig. 4 and Fig. 5) with the same total route and the same number of vehicles needed to serve the area, we can arbitrarily count either as the final solution. This example also illustrates the ability of the variability in the application of these methods to obtain multiple sub-optimal solutions, from which the optimal solution can then be selected based on additional information about the network and traffic on it, or other subjective factors of the solver. This may be the speed or continuity of the traffic flow, the saturation capacity of individual lines, etc.

## Analysis of methods for solving selected tasks

In the previous paragraphs, the possibilities of modelling different subproblems of VRP and LRP were pointed out. After considering the general requirements, the models implemented using heuristic methods were selected for the solution. Using the VRPTW model as an example, it was found that it is possible to achieve a suitable solution using heuristic algorithms, but it cannot be verified whether it is optimal. When the solution of the VRPB model is proposed, it can be seen that the solution found using the selected heuristic algorithms, although suitable, is not optimal. Heuristic methods are intuitive, they do not use exact mathematical algorithms, they try possible variants of the solution, and if it is better, they replace the current solution. The advantage of the given methods is the minimization of the computation time, which increases significantly when the requirements for finding a better solution are emphasized. Unlike heuristic methods, exact methods use exact algorithms to find the optimum and ensure that it is found.

## Conclusion

The impetus for the growth of the market for the transportation of people and goods using individual automobile transport vehicles, respectively light trucks, was the moment of the end of the COVID-19 crisis, during which a general slowdown in the movement of materials and goods and the mobility of citizens was recorded. In particular, the transportation of goods was carried out to a significant extent through individual and courier services at that time.

The issue of distribution of requests, whether people or freight, over a large network with a large number of traffic points (intersections) but also destination points of interest for service is characteristic especially for larger cities and agglomerations, where one or more vehicles serve multiple requests distributed over a managed network, which raises the problem of optimal routing of individual vehicles and overall fleet size. This also raises the issue of optimising investment costs - vehicle acquisition and operating costs - fuel, vehicle maintenance and repairs and other operating costs. The possibilities of solutions using graph theory show us the possibilities of modelling customer service using a minimum number of vehicles and therefore the possibility of minimising the necessary investment and operating costs. Thus, the use of graph theory methods gives us new possibilities to modify customer service algorithms with the possibility of optimizing operating costs.

The side effects of the optimal model of servicing the territory are, especially in urban networks and agglomerations, positive effects associated with fewer vehicles on the transport network, or their more efficient temporal and spatial management, which is associated with a lower incidence of congestion, fewer time delays, lower emissions of pollutants and greenhouse gases, as well as noise pollution for the affected population and areas.

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